Function Theory of a Complex Variable (E2): Exercise sheet 3

1. Determine all values of 2^i, i^i, $(-1)^{2i}$.

2. Define a single-valued branch of $\sqrt{1+z} + \sqrt{1-z}$ in a suitable region, and prove that it is analytic.

3. Compute
$$\int_\gamma x\,dz,$$
where γ is the line segment from 0 to $1+i$.

4. Compute
$$\int_{\partial B(0,2)} \frac{1}{z^2-1}\,dz,$$
where the circle is traversed anticlockwise.

5. Compute
$$\int_{\partial B(0,1)} \frac{e^z}{z}\,dz,$$
and
$$\int_{\partial B(2,1)} \frac{e^z}{z}\,dz,$$
where the circles are traversed anticlockwise.

6. Prove that if $f(z)$ is analytic in the whole of $\mathbb{C}$ and, for some integer n and $R < \infty$, satisfies
$$|f(z)| < |z|^n, \qquad \forall |z| \geq R,$$
then f is a polynomial.

7. If $f(z)$ is analytic for $|z| < 1$ and
$$|f(z)| \leq \frac{1}{1-|z|},$$
find the best estimate of $|f^{(n)}(0)|$ that Cauchy's inequality will yield.